Calculus III, Test 2 Review Key

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Note: If your decimal answers are close to mine, but not quite the same, that is fine. I have no problem with rounding error on my tests.

1. Calculate the limits:

   (a) $\lim_{t\to 0}\left(t^2\mathbf{i}+\frac{1-\cos t}{t}\mathbf{j}+\frac{3}{\ln t}\mathbf{k}\right)$ **Ans**: $\langle 0,0,0\rangle$.

   (b) $\lim_{(x,y)\to(0,0)}\frac{2x-y^2}{2x^2+y}$ **Ans**: Does not exist

2. Evaluate the definite integral $\int_0^2 (t\mathbf{i}+te^{t^2}\mathbf{j}-te^t\mathbf{k})\,dt$.

   Ans: $\langle 2,\frac{e^4-1}{2},-e^2-1\rangle$

3. A baseball player at second base throws a ball 90 feet to the player at first base. The ball is released at a point 5 feet above the ground with an initial velocity of 50 mph (which is 220/3 ft/sec) and an angle of 15° above horizontal. At what height does the player at first base catch the ball?

   Ans: 3.286 feet

4. Let $\mathbf{r}(t)=\sqrt{2}t\mathbf{i}+e^t\mathbf{j}+e^{-t}\mathbf{k}$. Find $\mathbf{T}(t)$, $\mathbf{N}(t)$, $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$ at $t=0$. Hint: simplify $||\mathbf{r}'(t)||$ before you find $\mathbf{N}$.

   Ans: $\mathbf{T}(t)=\frac{1}{e^t+e^{-t}}\langle\sqrt{2},e^t,-e^{-t}\rangle$, $\mathbf{T}(0)=\langle\sqrt{2}/2,.5,-.5\rangle$, $\mathbf{N}(0)=\langle 0,\sqrt{2}/2,\sqrt{2}/2\rangle$, $a_{\mathbf{T}}=0$ and $a_{\mathbf{N}}=\sqrt{2}$.

5. For $\mathbf{r}(t)=\sqrt{2}t\mathbf{i}+e^t\mathbf{j}+e^{-t}\mathbf{k}$ as in the previous problem, find the curvature K.

   Ans: In general, $K=\frac{\sqrt{2}}{(e^t+e^{-t})^2}$. At $t=0$ you get $\sqrt{2}/4$.

6. For $f(x,y)=\ln(xy-6)$, do the following:

   (a) Describe the domain and range. **Ans**: domain is all (x,y) such that $xy>6$. Range is all real numbers.

   (b) Sketch level curves at $c=-10, c=0$, and $c=2$ (you can use a graphing calculator and/or Sage to help with this). **Ans**:

   (c) Find $\nabla f(x,y)$. **Ans**: $\langle\frac{y}{xy-6},\frac{x}{xy-6}\rangle$.

   (d) On the sketch from part (b), plot $\nabla f(1,7)$ as a vector with its initial point at (1,7). What do you (or should you) notice?

   Ans: You should notice that the gradient vector is perpendicular to the level curve for $c=0$.

7. Discuss the continuity of the function $f(x,y) = -\frac{xy^2}{x^2+y^4}$. Where will it be continuous and where will it be discontinuous (if any)? Back up your assertions with mathematical reasoning. Graph f and explain how the graph reinforces your assertions.

   Ans: It will only be discontinuous at $(0,0)$ because the function is not defined there. It looks like the limit exists and equals zero if you come from the path of $x = 0$, $y = 0$, or $x = y$. If you come along the path $x = y^2$, though, you get a limit of -1/2, so the limit does not exist either at this point.

8. The formula for wind chill is given by

$$C = 35 + 0.6T - 36v^{0.16} + 0.4Tv^{0.16}$$

   where v is wind speed (in mph) and T is temperature in Fahrenheit. The wind speed is 23±3 mph and the temperature is $8^\circ \pm 1^\circ$. Calculate C at $(v,t) = (23,8)$ and use differentials to estimate the maximum propagated error for the given situation.

   Ans: error of ±2.4418 degrees.

9. Use partial derivatives to differentiate implicitly to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the equation

$$x + \sin(y+z) = 0.$$

   Ans: $\frac{\partial z}{\partial x} = \frac{-1}{\cos(y+z)}$ and $\frac{\partial z}{\partial y} = -1$.

10. Let $g(x,y) = 2xe^{y/x}$.

    (a) Find the direction of maximum increase at (x,y)=(2,0).

    Ans: $\langle 2,2\rangle$ is the gradient.

    (b) Find the slope if you were walking from (2,0) in the direction of the point (5,3).

    Ans: This is the same direction as the gradient, so you can either dot the direction vector or just find the norm of the gradient to get an answer of $2\sqrt{2}$.

11. Find the point(s) on the surface $z = 3x^2 + 2y^2 - 3x + 4y - 5$ at which the tangent plane is horizontal.

    Ans: $(1/2, -1, -31/4)$

12. Find the absolute extrema of $f(x,y) = 3x^2 + 2y^2 - 4y$ over the region in the xy-plane bounded by the graph of $y = x^2$ and $y = 4$.

    Ans: Abs. max of 28 at the point (2,4,28) and abs. min at the point (0,1,-2).